A Novel Effective Surface Impedance Formulation for Efficient Broadband Modeling of Lossy Thick Strip Conductors

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***Abstract* — A novel effective surface impedance is proposed for accurate modeling of the frequency-dependence of field penetration inside thick strip conductors. The proposed effective surface impedance is obtained through the rigorous modeling of the frequency-dependent cross-sectional field distribution in the interior of the lossy conductor, and is a function of both frequency and position along the perimeter of the cross section of the conductor. Availability of such an effective surface impedance enables the use of a surface integral equation formulation for the electromagnetic analysis of on-chip interconnects and integrated passives structures. Such formulations are much more efficient than volumetric ones without sacrificing accuracy in the modeling of the impact of conductor internal impedance and skin effect loss on the electromagnetic response. The validity of the proposed model is demonstrated through comparisons with measured scattering parameters for on-chip interconnect structures.**

I. Introduction

With device feature size continuing to shrink below the 0.1 micron regime, increased, mixed-signal functionality integration both on chip and in the package is aggressively pursued by circuit designers. Thus, the routine realization of system-on-chip (SoC) and system-in-package (SiP) designs is essentially limited only by the ability of computer-aided design tools to provide designers with he accuracy needed to tackle system design complexity as well as address the bottleneck associated with the testing of such high-density, multi-functional systems. A critical class of computer-aided design tools is the class of electromagnetic field solvers that are used not only for the analysis and design of integrated RF and microwave interconnects and passives but also for the accurate quantification of digital signal transmission and dispersion in high-speed digital interconnects. Furthermore, with on-chip clock frequencies in excess of 3 GHz today, aiming at the 10 GHz mark by the turn of the decade, there is a critical need for electromagnetic accuracy in the prediction of digital signal delay and crosstalk. Finally, with digital and multi-GHz analog circuitry in close proximity, electromagnetic interference effects require the use of electromagnetic field solvers for their proper modeling and accurate quantification.

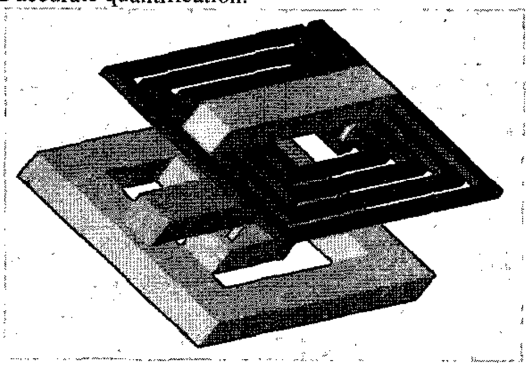

Fig. 1. Three-dimensional view of an on-chip integrated spiral inductor structure.

In summary, electromagnetic field solvers are fast becoming indispensable in the electrical performance analysis of high-speed digital, high-frequency analog and mixed-signal IC designs. However, the geometric and material attributes of these highly integrated designs pose new modeling challenges for the electromagnetic solvers used traditionally for planar microwave circuit analysis. Among them, one that is the focus of this paper is the proper modeling of thick conductors. Contrary to board-level applications where strip width-to-thickness ratios are in the order of 10, on-chip interconnects exhibit aspect ratios in the order of 1 or even smaller than 1 as the three-dimensional on-chip spiral geometry of Fig. 1 illustrates. Clearly, for such cases the field penetration inside the conductor must be modeled properly for accurate prediction of the electromagnetic response of the structure. To elaborate, assuming a square wire of conductivity 41 S/μm, conductor width of 1.0 μm, while the field distribution inside the wire is almost uniform at 1 GHz (where the skin depth is 2.49 μm), skin effect becomes well developed at a frequency of about 20 GHz. Furthermore, lateral coupling with adjacent wires is enhanced and needs to be modeled properly for accurate prediction of the electromagnetic response of a device, crosstalk calculation between interconnects, etc.

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To address this issue of accurate modeling of thick conductors several approaches have been proposed over the past few years. In the context of integral equation formulations in particular, and in addition to the brute-force volumetric formulation where the volume of the wire is discretized, thus resulting in a significant increase in the number of unknowns, several efforts have been reported toward the development of equivalent surface impedance formulations that capture correctly the field behavior inside the wire (e.g., [1]-[4]). The key advantage of such models is that the discretization of the volume of the conductor is avoided and, instead, a computationally more efficient surface integral equation formulation is used.

A novel effective surface impedance model is introduced in this paper. Direct and expedient broadband extraction of the effective surface impedance and its high accuracy are two of the key attributes of the new model that distinguish it from the ones presented in [1]-[4]. The development of the new model is presented in Section II. This is followed by a brief discussion of its implementation in a surface integral equation field solver, given in Section III. In Section IV a validation study of the new model is presented, followed by conclusions.

II. The Proposed Model

With the assumption that the current flow inside the conductor is parallel to the axis z of the conductor and with displacement currents assumed negligible compared to the conduction current, the governing equations for the electromagnetic field inside the conductor are,

$$\nabla^2 A_z(x,y) = -\mu J_z(x,y) \quad (1a)$$

$$J_z(x,y) = -\sigma \frac{dV}{dz} - j\omega\sigma A_z(x,y) \quad (1b)$$

Clearly, of relevance here is only the cross section of the conductor, which is assumed to be of arbitrary shape. In the following the subscript z, which indicates that the electric current density and the magnetic vector potential have only the axial component inside the conductor, will be dropped for simplicity. The first term on the right-hand side of (1b) may be interpreted as the impressed source. The solution of (1a) is expressed in terms of the integral statement,

$$A(x,y) = -\frac{\mu}{2\pi}\iint_S J(x',y')\ln\sqrt{(x-x')^2+(y-y')^2}\,dx'dy' \quad (2)$$

where S is the surface of the cross section of the conductor. Use of (2) in (1b) results in an integral equation for the unknown current density over the conductor cross section, the numerical solution of which is carried out through standard method of moments (MoM) procedures. With the choice of constant current density value over each element in the discrete model, the MoM approximation of the integral equation may be cast in the following matrix form,

$$\{J\} = \left([I]+s[B][D]\right)^{-1}\{J_{imp}\} \quad (3)$$

where $\{J\}$ is the vector of discrete unknown currents over each one of the segments in which the cross section is discretized, $\{J_{imp}\}$ is the vector with values proportional to the impressed source at each segment, [I] is the identity matrix and [D] is diagonal, while [B] depends on the properties of the material properties and the segment dimensions, and $s=j\omega$ is the complex angular frequency. It is noted that for the case of segments of rectangular shape the integral in (2) can be calculated in closed form. Furthermore, for enhanced modeling versatility, non-uniform meshing is implemented. Despite the non-uniformity of the mesh, the matrix [B] in (3) may be render symmetric through appropriate normalization. Consequently, its eigen-decomposition is straightforward, thus enabling the solution for the unknown currents to be cast in a pole-residue form with the complex angular frequency as independent variable [4]. As elaborated in [4], with a grid choice based on the accurate resolution of field penetration at the highest frequency of interest, the pole-residue form of (3) constitutes a frequency-dependent solution for the current distribution over the bandwidth of interest.

Once, the current density is known, the electric and magnetic field along the perimeter of the conductor can be obtained immediately as follows,

$$E_z(x_s,y_s) = -\frac{dV}{dz} + \frac{j\omega\mu}{2\pi}\cdot \sum_m J_z(x_m,y_m)\iint_{\Delta S_m}\ln\sqrt{(x_s-x')^2+(y_s-y')^2}\,dx'dy' \quad (4)$$

$$\vec{H}(x_s,y_s) = \mu^{-1}\nabla\times(A_z\hat{z}) = \frac{1}{4\pi}\sum_m J_z(x_m,y_m)\cdot \left[\iint_{\Delta S_m}\frac{-\hat{x}(y_s-y')+\hat{y}(x_s-x')}{(x_s-x')^2+(y_s-y')^2}\,dx'dy'\right] \quad (5)$$

In the above equations the subscript z has been re-instated for clarity. It is pointed out that in the case of rectangular elements the integrals in (4) and (5) can be obtained in closed form. The expressions in (4) and (5) lead directly to the definition of an effective surface impedance, Z_s^{eff}, at each point along the perimeter of the conductor as follows,

$$E_z(x_s,y_s)\hat{z} = Z_s^{eff}(x_s,y_s)\hat{n}\times\vec{H}(x_s,y_s) \quad (6)$$

where the unit vector $\hat{n}$, normal to the perimeter of the conductor, forms with the unit tangent, $\hat{t}$, along the perimeter, and the unit vector $\hat{z}$ a right-handed system at each point along the perimeter.

It is important to recognize the three key attributes of the derived effective surface impedance. First, it is position dependent, thus allowing for the impact of the cross-sectional shape to be modeled accurately. Second, it is localized, contrary to the global form of [4]. Finally, it is available as a continuous function of frequency over the bandwidth of interest. The implementation of (6) in an integral equation formulation will be discussed in the following section. However, prior to this a preliminary validation study of the proposed effective surface impedance model will be presented.

Table 1: Validation of effective surface impedance model

	From Z_s^{eff}		From internal fields	
f (GHz)	R_i (Ω/cm)	L_i (nH/cm)	R_i (Ω/cm)	L_i (nH/cm)
0.001	487.6	307.54290	[illegible]7.8	0[illegible].942856 53
1	487.6	407.54290	[illegible]7.8	1[illegible].342855 99
25	493.5	804.04211	[illegible]2.9	8[illegible].94252 4[illegible]

The validation is based on the observation that the internal impedance (per unit length), Z_i, for a lossy conductor may be obtained in two different ways. The first one makes use of well-known expressions for the per-unit-length resistance and internal inductance in terms of integrals of the square of the magnitude of the electric and magnetic fields respectively over the cross-section of the conductor [4]. Alternatively, the internal impedance can be obtained from the derived position-dependent effective surface impedance in terms of the following expression,

$$Z_i^{-1} = \left(R_i + j\omega L_i\right)^{-1} = \int_c \frac{dt}{Z_s^{eff}(t)} \qquad (7)$$

where the integral is along the perimeter of the cross section of the conductor. The comparison of the results obtained from these two approaches for a wire of rectangular cross section of width 1 μm, thickness 0.5 μm, and conductivity 41 S/μm is presented in Table I. Very good agreement is observed.

III. Implementation of Effective Surface Impedance

The derived effective surface impedance is easily implemented in a surface-based integral equation formulation of the electromagnetic field problem. For our purposes, the mixed potential integral equation (MPIE) is used. Since, the effective surface impedance is local, the MPIE statement at a point on the conductor surface becomes,

$$Z_s^{eff}(\bar{r})\bar{J}_S(\bar{r}) + j\omega \iint_{S_C} \bar{J}_S(\bar{r}')\overline{\overline{G_A}}(\bar{r},\bar{r}')ds' = -\nabla \iint_{S_C} \rho_S(\bar{r}')G_q(\bar{r},\bar{r}')ds' \qquad (8)$$

where the choice of the Green's function depends on the substrate properties.

For the purposes of this paper, the generalized Partial Element Equivalent Circuit (G-PEEC) interpretation of the (MPIE) was used for the development of the discrete approximation of (8) [5]. As a brief overview of G-PEEC it is mentioned that instead of the rectangular patches used by standard PEEC for the discretization of the conductor surface, G-PEEC allows for the use of the Rao-Wilton-Glisson (RWG) triangular patches [6] to enhance the modeling versatility of the discrete model.

IV. Numerical Results

As pointed out in the previous section, the position-dependent effective impedance enables the efficient modeling of the impact of the interior, frequency-dependent field distribution inside thick lossy conductors on the overall electromagnetic response of the system. Figures (2a) and (2b) depict the calculated effective surface impedances of a wire of rectangular cross section as a function of location around its perimeter at $f = 10$ GHz, with dimensions and conductivity those pertinent to the results in Table I. It is observed that as the frequency increases beyond 1GHz, the surface impedances on wide sides decrease, whereas the surface impedances on thick sides increase.

The proposed effective surface impedance was used in a G-PEEC-based solver to simulate the electromagnetic response of the meander-like on-chip coplanar waveguide test structure with ground straps, depicted in Fig. 3. The length of the line is about 5000 μm. Figure 4 depicts the simulated and measured scattering parameters. The test structures and measurements were provided by Mayo Clinic. Very good agreement is observed, especially in the frequency range of 0-15 GHz. Beyond 15 GHz, the measured data indicate slightly higher attenuation than the ones obtained from the model.

V. Conclusion

In summary, a new effective surface impedance formulation has been proposed for lossy conductors. The proposed formulation is streamlined for application to conducting wires used for planar interconnects and embedded passives, both on-chip and at the package level, of high-density ICs, for which the cross sectional aspect ratio (width to thickness) is often of the order of 1. The proposed effective surface impedance eliminates the need for discretizing the interior of the conductor, thus reducing the number of unknowns in the discrete models used for the electromagnetic modeling of the structures. The proposed effective surface impedance has the following three key attributes of the derived effective surface impedance. First, it is position dependent, thus allowing for the impact of the cross-sectional shape to be modeled accurately. Second, it is localized, and thus its implementation in numerical field solvers is simple and straightforward. Finally, it is available as a continuous function of frequency over the bandwidth of interest. Even though the derived effective surface impedance is compatible with both integral equation-based and finite method-based field solvers, the numerical experiments presented in this paper were based on the mixed potential integral equation formulation. Comparisons with measured scattering parameters for on-chip interconnect structures were used to demonstrate the validity and accuracy of the proposed effective surface impedance.

ACKNOWLEDGEMENT

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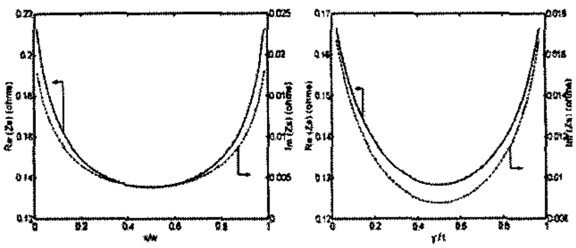
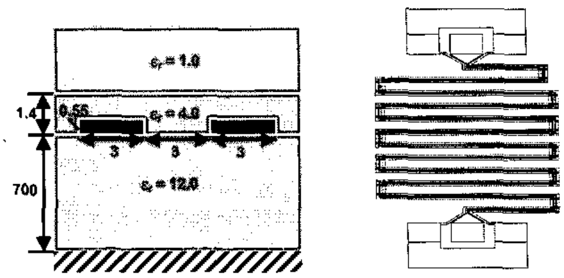


(a) Along width (b) Along thickness

Fig. 2. Calculated effective surface impedances of a wire of rectangular cross section as a function of position along its perimeter at 10 GHz.



(a) Cross section (b) Top view

Fig. 3. Meander-like on-chip coplanar waveguide with ground straps at corners (dimensions in microns)

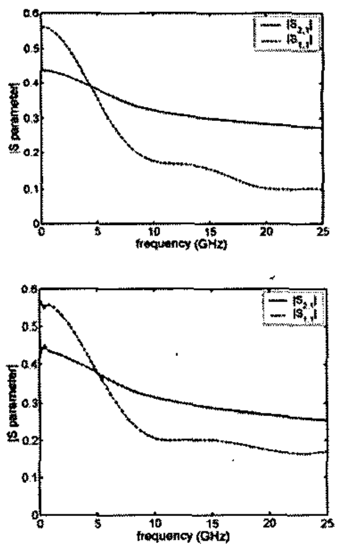


Fig. 4. Scattering parameters of meander-like on-chip coplanar waveguide (top: calculated; bottom: measured).